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THE STEADY AND TRANSIENT FREE CONVECTION BOUNDARY LAYER ON A UNIFORMLY HEATED VERTICAL PLATE

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Abstract—A Zehnder–Mach interferometer was used to study the free convection thermal boundary layer about a uniformly heated vertical plate and to derive the heat transfer coefficients connected with this situation. The experiments were performed when the plate was immersed in water and the steady state boundary layer, as well as its transient development from an initial state at rest and with uniform temperature to steady state condition, was investigated when a step function in the power input to the plate was applied. Results for the steady state runs agree very well with the results of an analysis by Sparrow and Gregg. The transient runs indicate that the temperature field in the fluid develops initially in the same way as for heat conduction into a semi-infinite solid. After a short transition period, the steady state condition is reached. The boundary layer grows with time in such a way that it increases at first with increasing time, reaches a maximum, and decreases again until it settles to its steady state value. The wall temperature and the local heat transfer coefficient can be predicted for the whole period from start to steady state by the solution for one-dimensional unsteady conduction or for the steady state boundary layer.

Résumé—Un interféromètre de Zehnder–Mach a été utilisé pour étudier la couche limite thermique de convection libre sur une plaque verticale uniformément chauffé et pour déterminer les coefficients de transmission de chaleur relatifs à cette disposition. Dans ces expériences, la plaque était immergée dans l'eau et la couche limite en régime permanent, ainsi que son développement transitoire à partir d'un état de repos initial et d'une température constante, a été étudiée quand on applique à la plaque une augmentation échelon de la puissance de chauffage.

Les résultats pour le régime permanent concordent très bien avec ceux d'une étude de Sparrow et Gregg. En régime transitoire, le champ de température évolue de la même façon que la conduction dans un solide semi-infini. Après une courte période de transition la condition du régime permanent est atteinte. La couche limite évolue avec le temps de façon à s'épaissir dans les premiers instants, elle atteint ensuite un maximum, puis décroit jusqu'à sa valeur du régime permanent. La température de paroi et le coefficient local de transfert de chaleur peuvent être calculés, pour la période allant du départ de l'expérience jusqu'au régime permanent, en utilisant la solution de la conduction non permenente unidimensionnelle ou de la couche limite permanente.

Zusammenfassung—Mit Hilfe eines Zehnder–Mach-Interferometers wurde die thermische Grenzschicht bei freier Konvektion an einer gleichförmig geheizten senkrechten Platte untersucht und daraus die Wärmeübergangskoeffizienten abgeleitet. Die Platte war bei den Versuchen in Wasser getaucht. Die stationäre Grenzschicht und ihre Entwicklung aus einem Anfangszustand in Ruhe mit gleichförmiger Temperatur zum stationären Zustand wurden bei schrittweiser Änderung der Heizleistung der Platte untersucht. Die Ergebnisse der stationären Versuche stimmen gut mit den Rechenergebnissen von Sparrow und Gregg überein. Die nichtstationären Versuche zeigen, daß sich das Temperaturfeld in der Flüssigkeit anfänglich wie bei der Wärmeleitung in einen halb-unendlichen Körper entwickelt. Nach einer kurzen Übergangsperiode wird der stationäre Zustand erreicht. Die Grenzschichtdicke steigt anfänglich mit der Zeit an und fällt nach Erreichen eines Maximums auf ihren stationären Wert ab. Die Wandtemperatur und die örtlichen Wärmeübergangskoeffizienten können für die ganze Periode vom Start bis zum stationären Zustand durch die Lösungen für die eindimensionale nichtstationäre Wärmeleitung oder für die stationäre Grenzschicht vorausgesagt werden.

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Abstact—Для изучения теплового пограничного слоя на равномерно нагретой вертикальной плите в условиях свободной конвекции и для получения соответствующих коэффициентов переноса тепла использовался интерферометр Маха-Цендера. Были проведены опыты с погружённой в воду плитой и исследован пограничный слой как установившийся, так и в переходном режиме от начального состояния покоя и одинаковой температуры к стационарному состоянию при ступенчатом подводе энергии к плите. Результаты для установившихся потоков хорошо согласуются с результатами анализа Грегга и Спэрроу. Неустановившиеся потоки означают, что температурное поле в жидкости (газе) развивается так же, как и в теплопроводящем полуограниченном твердом теле. После короткого переходного периода достигается установившееся состояние. Пограничный слой изменяется со временем следующим образом: сперва он растёт, достигает максимума и уменьшается до тех пор пока не достигает величины, отвечающей установившемуся состоянию. Температура стенки и локальный коэффициент теплообмена могут быть получены для всего периода, от начала до установившегося состояния, с помощью решения для одномерной нестационарной теплопроводности или для установившегося пограничного слоя.

NOMENCLATURE

g = gravitational acceleration;
h = local heat transfer coefficient;
k = heat conductivity;
q = heat flow through plate surface per unit time and area;
x = distance from lower plate end;
y = distance from plate surface;
a = thermal diffusivity;
β = thermal expansion coefficient;
ϑ = difference between temperature in boundary layer and in bulk of fluid;
ϑ_w = difference between plate surface temperature and fluid bulk temperature;
τ = time;
ν = kinematic viscosity;
$Gr = \frac{g\beta\vartheta_w x^3}{\nu^2}$ Grashof number;
$Nu = \frac{h\,x}{k}$ Nusselt number;
$Pr = \frac{\nu}{a}$ Prandtl number.

INTRODUCTION

STUDIES of convective heat transfer were, in the past, restricted almost exclusively to steady state conditions. This was justified because the heat capacity of boundary layers is generally so small that steady state relations describe heat transfer coefficients with sufficient accuracy even in situations where the temperatures at the surface and outside the boundary layer vary with time. Such a situation is commonly referred to as "quasi steady". Only recent developments in engineering have led to conditions under which temperatures vary so rapidly that convective heat transfer coefficients become significantly different from the steady values. As a consequence, analytical studies of unsteady boundary layers have been published in various magazines. Experimental information on unsteady boundary layers, however, is still very restricted.

Unsteady free convection boundary layers are of special interest since they are generally thicker and have, therefore, a larger heat capacity than boundary layers in forced flow. As a consequence, heat transfer coefficients differ from the steady state values already at relatively small rates of temperature variations with time. Such unsteady free convection boundary layers arise, for instance, in nuclear reactors when the pumps for the cooling system fail or with power fluctuations in reactors cooled by free convection.

The earliest analysis of a free convection unsteady boundary layer on a flat plate, infinite in extent, has been performed by Illingworth [1]. The temperature field in the boundary layer was found to be identical with the temperature field for unsteady one-dimensional heat conduction into a semi-infinite body. Sugawara and Michiyoshi [2] included in their analysis conditions near the leading edge of the plate. Their calculation mainly covers the very early stages of the boundary layer growth. Siegel [3] investigated analytically the boundary layer growth up to the time when steady state is obtained for the condition that the surface temperature or the surface heat flux changes in a step-wise fashion from zero to a finite value. His results are of special interest in the present study and will be discussed in more detail later on.

Ostroumov [4] and Gillam *et al.* [5] performed experimental studies of transient free convection existing on a horizontal wire. The measurements were essentially restricted to a determination of the wire temperature and of the heat flux to the surrounding fluid as a function of time. Klei [6] performed similar experiments on a 0·0005 in. thick and 1 in. high platinum foil which was suddenly heated by an electric current. McLean *et al.* [7] made a study on a horizontal nichrome foil in a water tank, using a Zehnder–Mach interferometer. In this last study interest was restricted to the early stages of the boundary layer development.

The present paper describes an experimental investigation of an unsteady free convection boundary layer on a vertical flat plate under the condition that the heat flux at the surface is locally constant and changes in a step-wise fashion with time. Complete information on the temperature field in the boundary layer as a function of time has been obtained by use of a Zehnder–Mach interferometer and the study includes the whole range from the beginning of the boundary layer development to the steady state condition. The experiments were originally carried out in air. It was found, however, that the above boundary conditions could be obtained in a much cleaner way by a study in water, and the results reported in this paper will be restricted to this fluid. The fact that the change in optical refraction coefficient of water with temperature is much larger than that of air made it necessary to carry out the experiments with very small temperature differences. This, however, turned out to be an advantage, because the results of these experiments can well be compared with analytical results which have all been obtained for a fluid of constant properties. The study of free convection in water makes it also possible to investigate heat transfer to a fluid with internal heat sources. The internal heat generation can easily be obtained when the water is made slightly conducting and when an electric current is directed through it. A brief study of this kind was reported by Goldstein [8].

EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION

The main requirement for the heated vertical plate is that it has a heat capacity which is as small as possible so that it can follow time-wise variations of temperature readily. For this purpose a 0·001 in. thick foil of 302 stainless steel was used. The foil was stretched tightly between two horizontal straight edges by six screws on each end of the foil. This can be recognized in Fig. 1. Two horizontal straight edges *a* are fastened to an aluminum plate *b* and this plate in turn is connected through the rods *c* and a bridge *d* shown in the figure with the frame of the interferometer. The vertical part of the foil *e* between the two edges which represents the heated vertical plate has a dimension of 4·01 in. in horizontal and 6·503 in. in vertical direction.

The foil was heated by passing an electric current through it. For this purpose, two strips *f* of copper, 0·005 in. thick, were soldered to the upper and lower end of the foil. The strip at the lower end was moved as closely as possible to the location of the straight edge so that the heated part of the foil started very close to the point where it bent into the vertical direction. The copper strips in turn were connected by heavy conducting cables to three storage batteries connected in parallel. A bank of resistors in series with the foil allowed adjustments in the internal heat generation in the foil. A precision shunt and a calibrated millivoltmeter (Weston Model 1) measured the current and a similar millivoltmeter measured the potential drop in the foil. A hand-operated switch turned the current on and off. The uniformity of the heated foil was checked by a super-micrometer and by weighing small pieces cut out of the sheet. No variations greater than 1 per cent in local thickness and weight were found. The electric resistance of the sheet was 0·0431 Ω. The uniformity of the foil guaranteed a locally uniform heat generation by the electric current.

The heated vertical plate was put into a prismatic water tank as shown in Fig. 2. The tank *a* had the dimensions 12 × 14½ in. in a horizontal plane and a height of 24 in. The most important elements of the tank were the two windows which were required in order to pass the light

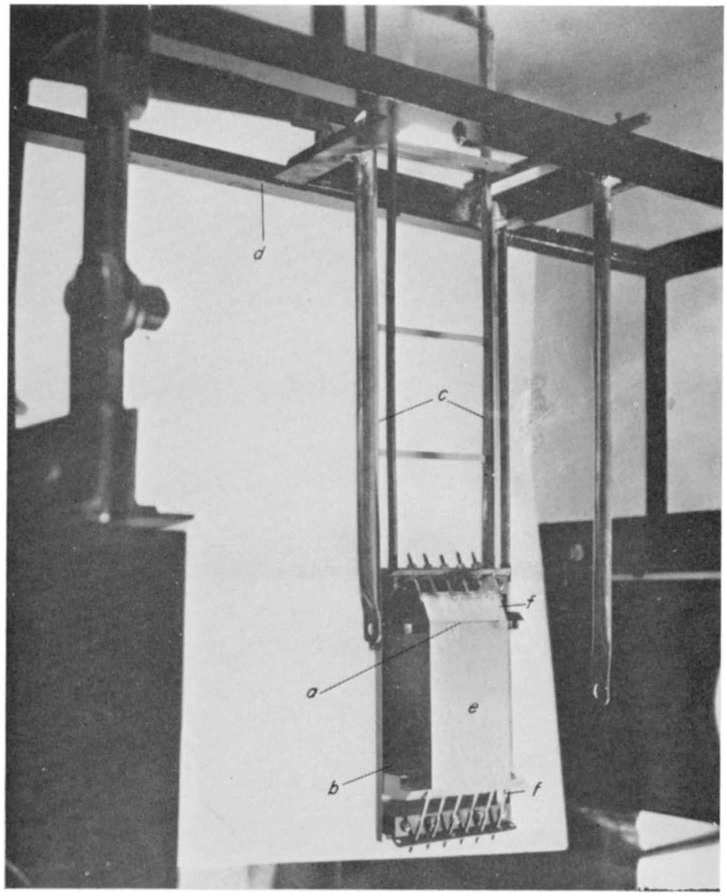

FIG. 1. Heated sheet and holder: *a*, horizontal straight edges; *b*, aluminum plate; *c*, connecting rods; *d*, bridge construction; *e*, heated foil; *f*, copper strips.

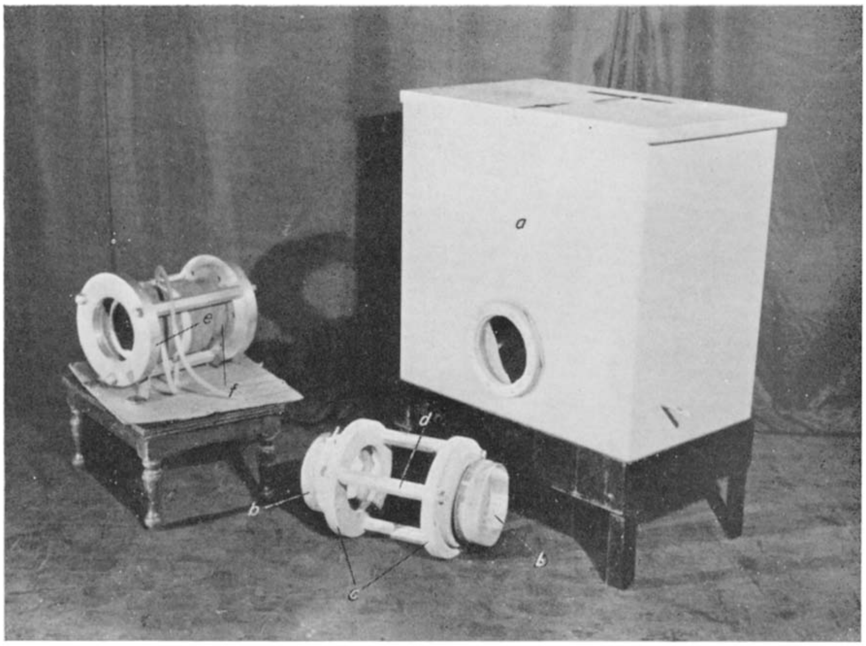

FIG. 2. **Water tank, compensating tank, and window support frame for main tank:** *a*, main water tank; ***b*, glass windows**; *c*, aluminum rings; *d*, aluminum rods; ***e*, compensating tank; *f*, rubber inner tube.**

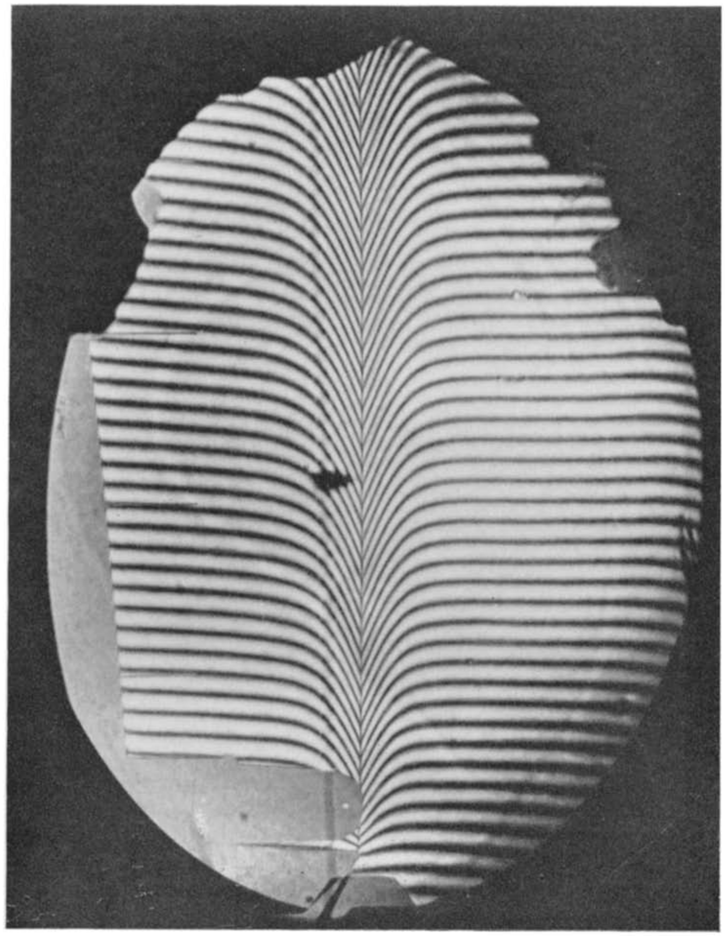

FIG. 3. Interferogram of heated foil with boundary layers.

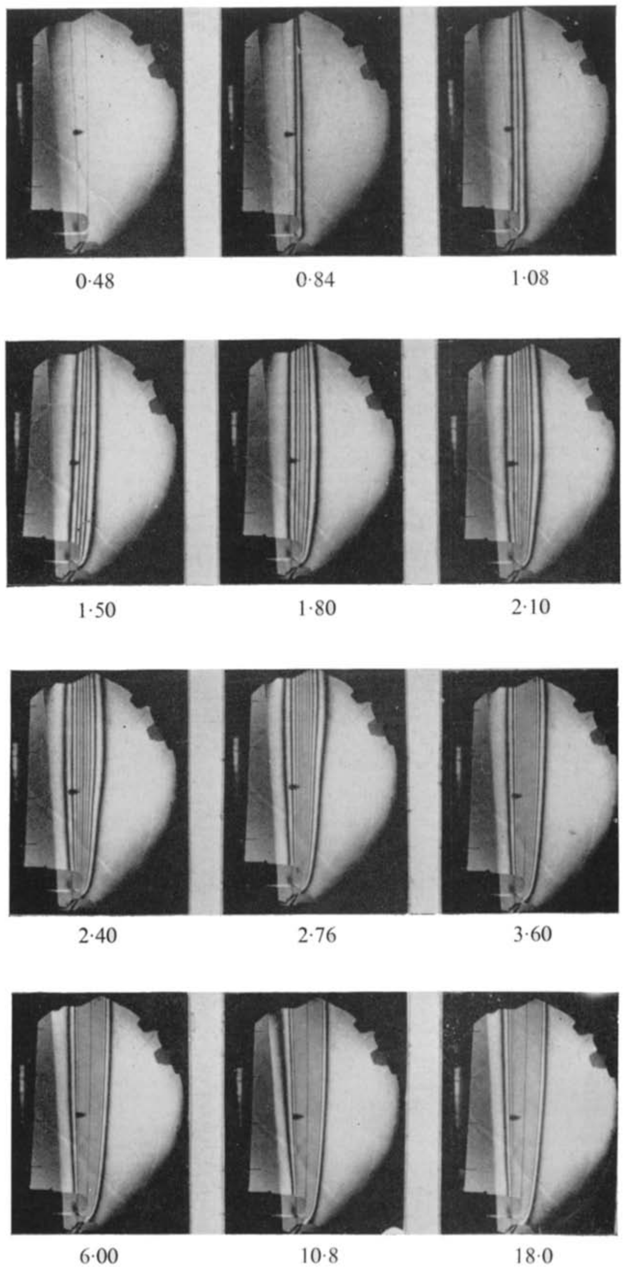

FIG. 8. Interferograms of the unsteady free convection boundary layer. The numbers under the photos give the time from the start of the heat pulse in seconds.

beam of a Zehnder–Mach interferometer past the heated plate. The two windows *b* were made of plate glass, 1 in. thick and 6 in. in diameter, and polished so that each surface was flat to one-fourth wavelength. Each plate was checked individually for the absence of schlieren by placing it into the light beam of the interferometer. The two windows were held 8 in. apart in a special frame which can be recognized in Fig. 2 arranged in front of the tank. The glass plates were fastened between two aluminum rings *c* and these in turn were held in the proper distance by three hexagonal aluminum rods *d* with screws at both ends. This frame was attached by the three vertical rods *g*, which can be seen in Fig. 1, to the interferometer frame. The space between the aluminum disks which held the windows and the walls of the tank was sealed off by two pieces cut from a rubber inner tube. The design of the window frame was made in this way to enable the experimenter to adjust the two windows so that they are parallel to each other and normal to the light beam direction and also to connect them rigidly with the interferometer.

The tank was filled with distilled water and this filling was replaced several times during the experiments. A platinum resistance thermometer and a Mueller bridge were used to determine the bulk water temperature in the tank. This thermometer was calibrated with a second resistance thermometer whose calibration had been obtained from the National Bureau of Standards.

The presence of the two glass windows and of the water in one of the light beams of the interferometer increases the optical path length of this beam. A compensating tank *e* was therefore arranged in the second light beam. This tank was similar in design to the window frame in the main tank. It is shown on the left-hand side in Fig. 2. A piece of rubber inner tube *f* connected the aluminum rings which held the glass windows. The interior of the tube was filled with distilled water. The windows themselves were of the same optical quality as the ones in the main tank.

A Zehnder–Mach interferometer was the main tool in these experiments. This instrument was constructed at the Heat Transfer Laboratory. It is very similar in design to one which had been built by Eckert *et al.* [9]. The basic system of the interferometer consists of four properly coated plates of optical glass with 6 in. diameter spaced at the corners of a rectangle with 55 and 27·5 in. side lengths. The opening of the interferometer in which the test equipment can be arranged is 40 in. in light beam direction. A detailed description of the interferometer and its operation is contained in [9] and will not be repeated here. The main difference in the instrument used in the present tests and the one described in the reference lies in the fact that in the Heat Transfer Laboratory Interferometer the optical path in one of the two beams can be changed by a lateral movement of one mirror. Fig. 3 shows an interferogram as it is obtained in the interferometer. This specific photo is from the initial series in which the experiments were carried out in air. Corresponding photos were obtained in the water runs; they are, however, less suitable for reproduction. It can be observed that the interferometer was, in the experiments, adjusted in such a way that horizontal fringes appear in the interferogram when the light beam travels through a field of locally uniform density. In the heated boundary layer, the fringes are bent in a downward direction. The image of the heated vertical foil can also be recognized. For the tests in water, the difference between the temperature of the heated foil and the bulk temperature of the fluid outside the boundary layer was kept to values below 3°F since considerably larger temperature differences caused the interference fringes to bend excessively and in this way to become indistinguishable. The small temperature difference made it necessary to keep the fluid bulk temperature very uniform and constant during the duration of an experiment. For this reason, the water tank was well insulated and the bulk temperature of the water was kept to a value close to room temperature. A maximum rate of change of the water temperature of 0·0005°C/min was allowed during the experiments. This results in a fringe shift of 0·1 fringe in 5 min. For the transient runs, a movie camera was mounted with its electric drive mechanism on the interferometer frame. An electric clock timer was arranged so that it appeared on the film simultaneously with the interferogram.

The evaluation procedure of the interference photos is described in detail in [8]. Two main errors have to be considered if a temperature field is determined by evaluation of an interferogram. A refraction error will arise as a consequence of the fact that the rays in the light beam are slightly bent from their original direction when the beam travels parallel to the plate surface through the boundary layer, because a transverse temperature gradient exists in the boundary layer. This error was calculated following the procedure described in [10] and was found to be less than ½ per cent during all steady state runs. During the very early part of the transient runs when the boundary layer is very thin, the error can be considerably larger. This, however, is not the main time of interest. A second error is caused by the fact that the temperature field does not end abruptly in the two planes containing the two vertical ends of the plate and normal to it, but extends somewhat beyond the plate length.

For a thin plate within which uniform heat generation is taking place, there should be no first-order error in the average temperature gradient in the fluid at the plate surface due to end-effects. This follows from the effective integration of the temperature field by the light beam and from the fact that, with the present symmetry, all the heat must leave the plane of the plate. The validity of this reasoning is confirmed by the good agreement between the measured heat flux and the electrical power input. A calculation [8] similar to that of Eckert and Soehngen [11], indicated a small (approximately 1 per cent) positive error in the plate temperature due to end-effects. This holds for a plate whose temperature is uniform. With the actual system, the temperature at the edges tends to fall off slightly, compensating this error, and it was assumed that the results could be used directly. Confirmation of this comes from the good agreement with available analytical solutions.

The properties of water which are required for an evaluation of the interferograms and of the dimensionless parameters in which the test results are presented, have been taken from the literature. Values of the index of refraction for water are contained in a paper by Tilton and Taylor [12]. They were also checked in the following way: The water in the main tank was heated to various temperature levels and the shifts of the zero interference fringe corresponding to the step-wise temperature increases were recorded. The change of the index of refraction with temperature could then be readily calculated from the fringe shifts. Only this value is needed for the evaluation of the interferograms. The measured values agreed within ± 2 per cent with those taken from the literature.

In any temperature boundary layer, the problem arises at what reference temperature the properties should be inserted into the dimensionless parameters which describe the process. In the present experiments this question is relatively insignificant because of the small temperature differences existing within the boundary layer. A film temperature half-way between the wall and the fluid bulk temperature was used as reference temperature.

PRESENTATION AND DISCUSSION OF RESULTS

Two groups of experiments were performed. In the first one, the boundary layer characteristics and heat transfer were investigated for steady state condition. The second group dealt with the transient development of the free convection boundary layer.

Steady state studies

Figs. 4 to 7 present the results of the experiments for steady state. In Fig. 4, the heat flow q from the surface of the heated plate into the water per unit area and time is plotted over the distance x from the leading edge measured in

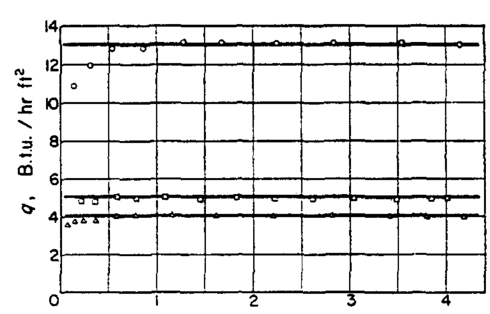

FIG. 4. Heat flow q from surface of heated plate by steady free convection as obtained from electric measurements and from interferogram.

vertical direction along the plate surface. Three runs are evaluated which differ by the amount of current passed through the plate. The horizontal lines indicate the electric power converted into heat per unit surface area. These values were obtained from the readings of the amperemeter and the voltmeter. The points in Fig. 4 have been obtained from the evaluation of the interferograms. The temperature gradient at the plate surface determined from the shift of the interference fringes was multiplied with the heat conductivity of water at the plate surface temperature. The agreement between heat generation in the plate and the heat flux through the plate surface is excellent except for a small region near the leading edge. Apart from this region the condition of a locally constant heat flux is therefore satisfactorily fulfilled.

Fig. 5 presents the temperature ϑ_w of the plate surface as measured above the bulk temperature of the water, again plotted over the distance x.

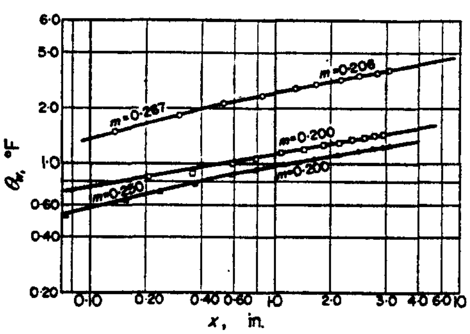

FIG. 5. Temperature variation along the height x of the heated plate for steady free convection. Three experiments with the heat fluxes indicated in Fig. 4 are presented in this and the following figure; ϑ_w = temperature difference between wall surface and fluid bulk; m = exponent expressing the temperature variation.

The temperature of the plate surface was again determined from the interferograms. In this connection it proved advantageous that two symmetrical boundary layers on both sides of the foil representing the plate could be observed. The temperature profiles obtained from the fringe shifts on both sides of the plate were extrapolated to the plate center line. The temperature indicated by the profile at a distance from the center line equal to half the plate thickness was then interpreted as the plate surface temperature. It may be observed in the figure that this plate temperature increases proportional to a power $m = 0{\cdot}200$ to $0{\cdot}206$ of the distance x when the region near the plate leading edge is excluded. This is in excellent agreement with the analysis by Sparrow and Gregg [13] which predicts that the surface temperature of a vertical plate in steady free convection flow increases proportionally to the one-fifth power of the distance x when a locally constant heat flux at the plate surface is prescribed. For the region near the leading edge, a constant heat flux would lead to very steep temperature gradients. Heat conduction in the plate along the direction x will tend to reduce these temperature differences and to decrease the temperatures as compared with those for constant heat flux. This conclusion agrees with the results presented in Fig. 5.

Fig. 6 presents the local Nusselt number Nu plotted over the parameter Grashof times Prandtl. The points are again the results obtained from the interferograms, the full line presents the results of the analysis [13]. The product $GrPr$ has been selected because the Nusselt number is almost a function of $GrPr$ only

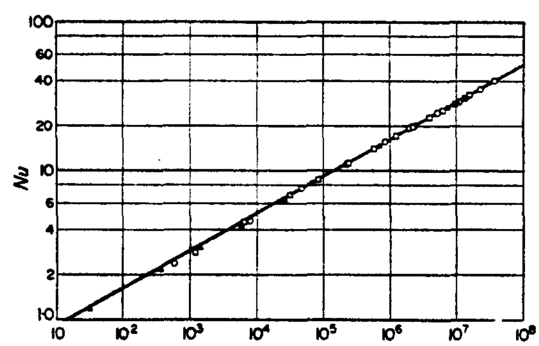

FIG. 6. Nusselt number for steady free convection as a function of the product Grashof times Prandtl number.

	q (B.t.u./hr ft²)	t_∞ (°F)
○	130·2	72·64
□	50·8	73·51
△	40·8	72·64

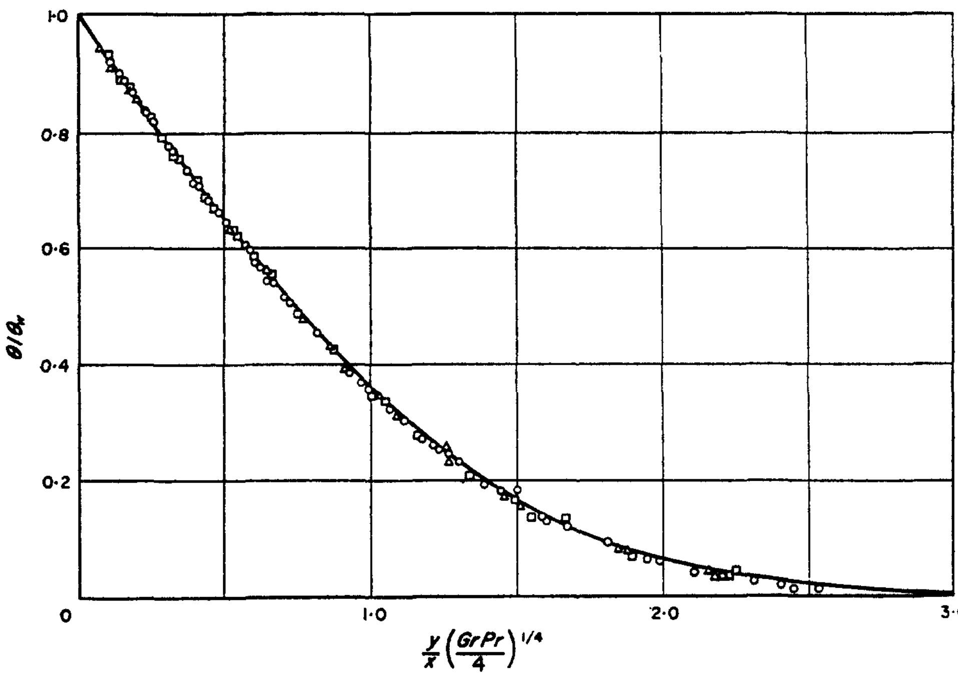

FIG. 7. Temperature profile in the steady free convection boundary layer. ϑ = temperature difference between a location at distance y from the plate and the fluid bulk. ϑ_w = temperature difference between the plate and the fluid bulk. x = distance from the lower end of the foil.

	q (B.t.u./hr ft^2)	t_∞ (°F)
○	130·2	72·64
□	50·8	73·51
△	40·8	72·64

according to analysis. In this way small changes of the bulk temperature of the water which occurred between the individual tests are suppressed in the figure. It may be observed in Fig. 6 that the agreement between measured and analytical results is excellent. Using the experimental points for the distance x, for which the heat flow is locally constant, an average value of 0·513 with a standard deviation of ½ per cent is obtained for the parameter $Nu/(GrPr)^{1/4}$. This compares with the value 0·5146 resulting from the analysis.

Fig. 7 shows the temperature profile within the boundary layer in a dimensionless presentation. The ratio of the local temperature ϑ within the boundary layer to the plate surface temperature ϑ_w, both measured above the fluid bulk temperature, is plotted over a dimensionless wall distance parameter as it has evolved from the analysis. The experimental points can be compared with the full line which represents the results of the analysis by Sparrow and Gregg for a Prandtl number 6·4.* It may be observed that the agreement between the calculated and measured temperature profiles is excellent.

Transient studies

Fig. 8 shows a series of interferograms obtained in the transient studies. These photos were taken

* The results for Pr = 6·4 are not contained in [13]. They were calculated for us by the authors, which is gratefully acknowledged.

when the heated plate was surrounded by air. The adjustment of the interferometer was such that no interference lines could be observed in a field of constant temperature. The interference fringes within the heated boundary layer are then practically lines of constant temperature. The figures below the individual photos give the time from the start of the experiment. Similar photos have been obtained from the tests with water as the fluid surrounding the plate. These photos, however, are less suitable for reproduction.

Fig. 8 indicates that the thermal boundary layer grows at first with locally uniform thickness. In other words, the boundary layer thickness is originally independent of the location x. Later on the influence of the lower plate edge makes itself felt by the fact that the boundary layer stays thinner in the region near this edge. The position on the sheet up to which the boundary layer is a function of x moves gradually upward with increasing time.

These conclusions, obtained from an observation of the interferograms, are substantiated by the evaluations which are presented in the Figs. 9 through 12. Fig. 9 presents the heat flux q per unit surface area in unit time as a function of the time measured from the moment where the electric current is turned on. Two runs with different heating rates and four positions along the heated plate have been evaluated. The points are again obtained from the interferograms and the full line from the electric measurements. It may be observed from the figure that, a few seconds after the initiation of the heat generation, the heat flux from the surface of the plate is independent of time as well as of location.

Fig. 10 presents the surface temperature ϑ_w of the heated plate as a function of time for four locations x along the plate and for two different heating rates. The figure indicates clearly that for a considerable period the wall temperature is independent of location. After a short transition time it becomes independent of time, but is now dependent on the distance x. For the second period, therefore, steady state has been reached. The transition from the first to the second period is rather short and occurs at increasing times for increasing distance x.

Illingworth [1] has shown that the unsteady free convection temperature boundary layer in front of an infinite plate is described by the equation which represents unsteady one-dimensional heat conduction into a semi-infinite body. He considered a step in the wall temperature. For a step in the heat flux, the following equation describes, according to [14],

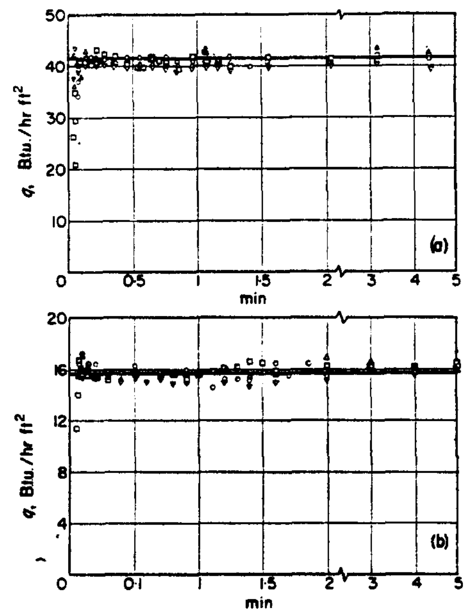

FIG. 9. Heat flux at the surface of the plate as a function of the time from the start of the heat pulse for two different heat fluxes.
$\nabla x = 0{\cdot}32$ in. $\triangle$ 1·03 in. $\bigcirc$ 2·10 in. $\square$ 3·80 in.

the temperature field for unsteady heat conduction in a semi-infinite wall

$$\vartheta = q\,\frac{\sqrt{(a\tau)}}{k}\left\{\frac{2}{\sqrt{\pi}}\exp\left(\frac{-y^2}{4a\tau}\right) - \frac{y}{\sqrt{(a\tau)}}\operatorname{erfc}\frac{y}{2\sqrt{(a\tau)}}\right\} \quad (1)$$

The wall temperature is given by the relation

$$\vartheta_w = q\,\frac{2}{k}\sqrt{\left(\frac{a\tau}{\pi}\right)} \quad (2)$$

and the heat transfer coefficient by the equation

$$h = \frac{k}{2}\sqrt{\left(\frac{\pi}{a\tau}\right)} \quad (3)$$

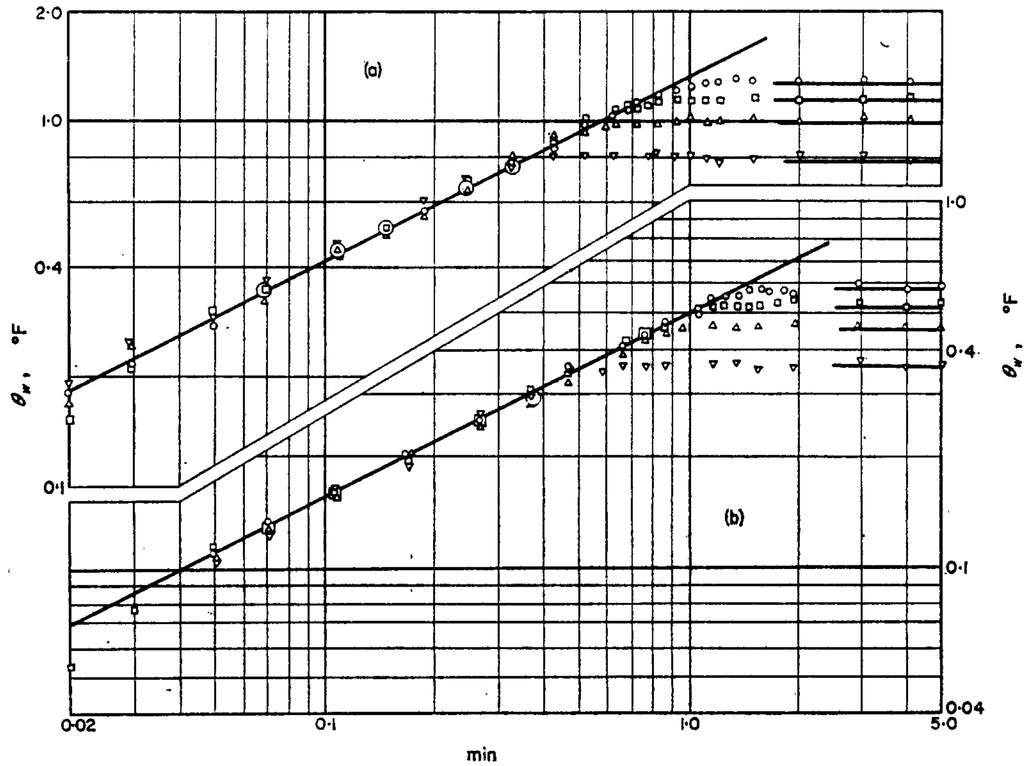

FIG. 10. Temperature increase of the heated plate as a function of time from the start of the heat pulse. ϑ_w temperature difference between the plate surface and the fluid bulk. The notation (a) and (b) in this and the following figures corresponds to the one in Fig. 9.
$\nabla x = 0{\cdot}32$ in. $\triangle$ 1·03 in. $\square$ 2·10 in. $\bigcirc$ 3·80 in.

The relation expressed by equation (2) is inserted in Fig. 10 as the heavy inclined line. It can be observed that the measured points agree closely with this analytical solution, indicating that, in the initial stages, an unsteady free convection boundary layer grows on a finite plate in the same way as on one which is infinite in extent. The solution obtained by Sparrow and Gregg [13] for a steady state free convection boundary layer on a vertical plate with constant heat flux through its surface can be expressed in the following form for ($Pr \approx 6{\cdot}4$):

$$Nu = 0{\cdot}726 \left(\frac{GrPr}{4}\right)^{1/4} \qquad (4)$$

The plate surface temperature as obtained from this equation and from the condition that the heat flow is independent of location is

$$\vartheta_w = x^{1/5} \left(\frac{q\sqrt{(2)}}{0{\cdot}726k}\right)^{4/5} \left(\frac{\nu\alpha}{g\beta}\right)^{1/5} \qquad (5)$$

The values of the wall temperature calculated from this equation for the condition existing in the experiments are indicated by the horizontal lines in Fig. 10. The agreement with the experimental points is very good.

Fig. 11 shows the heat transfer coefficient as a function of time. The inclined line gives the heat conduction solution (equation (3)) and the short horizontal lines represent the heat transfer coefficients obtained from equation (4). From the Figs. 10 and 11, the following rule for a calculation of the wall temperature and of the local heat transfer coefficients in an unsteady free convection boundary layer on a vertical plate can be derived. One calculates both parameters for the initial conduction phase as

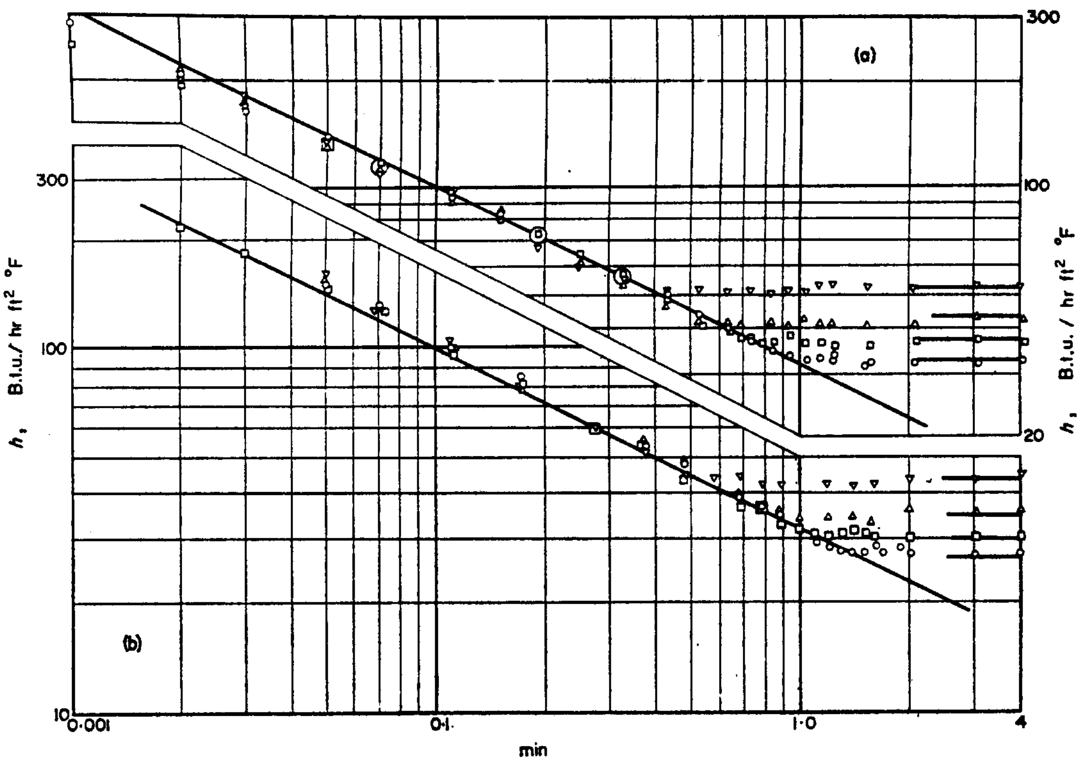

FIG. 11. Local heat transfer coefficient h at the plate surface as a function of time from the start of the heat pulse.
$\nabla x = 0{\cdot}32$ in. $\triangle$ 1·03 in. $\square$ 2·10 in. $\bigcirc$ 3·80 in.

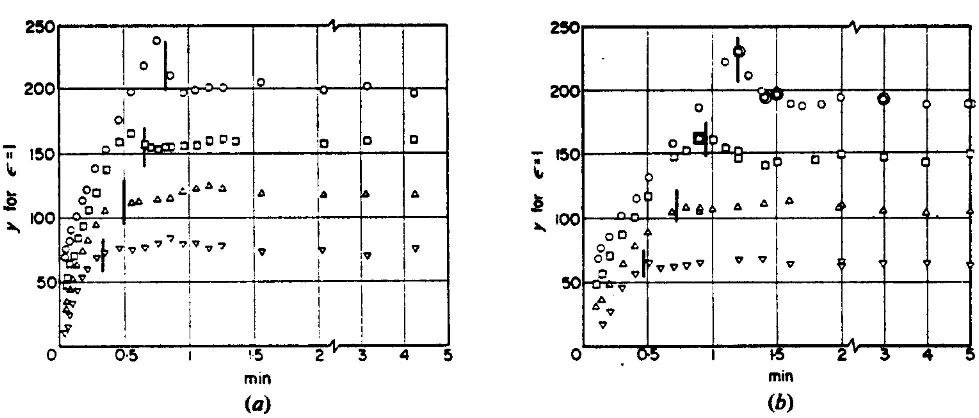

FIG. 12. Wall distance y, at which the temperature increase caused one fringe shift in the interferogram, as a function of time from the start of the heat pulse.
$\nabla x = 0{\cdot}32$ in. $\triangle$ 1·03 in. $\square$ 2·10 in. $\bigcirc$ 3·80 in.

well as for steady state condition. The smaller one of the two temperatures describes the actual wall temperature satisfactorily and the larger one of the two heat transfer coefficients gives the local heat transfer coefficient with good accuracy.

Siegel [3] performed an analysis of the unsteady free convection boundary layer on a vertical plate. He used for this purpose the integrated momentum and energy boundary layer equations together with plausible assumptions on the shape of the velocity and temperature profiles. On the basis of his analysis, he predicted that the boundary layer grows initially in thickness with increasing time; that it reaches a maximum, and decreases again before it settles to its steady state value. Correspondingly, the heat transfer coefficient is predicted to decrease initially, to pass through a minimum and then to increase again to its steady state value. The heat transfer coefficients, as presented in Fig. 11, appear to indicate such a minimum. It is, however, very slight and possibly within the accuracy of the measurements. In order to compare the predicted growth of the boundary layer thickness with the measured one, the following procedure was used: the distance from the plate at which the temperature has increased by such an amount that it caused one fringe shift was measured in the interferograms and is plotted in Fig. 12, again for two heating rates and for four positions x. Siegel's prediction that the boundary layer development with time goes through a maximum is, according to this figure, well verified. The vertical lines in Fig. 12 indicate the time for which, according to the analysis, the pure conduction period ends. These times again agree remarkably well with the times at which the maximum of the boundary layer thickness occurs. The fact that this maximum is not accompanied by an equally pronounced minimum in the heat transfer coefficient must obviously indicate that the shape of the temperature profile changes during the transition period from pure conduction to steady state.

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